# Advanced Theoretical Physics

## A Historical Perspective

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## 7.2 Spacetime

When a physics student first learns about special relativity, abstract equations are often thrown at them with little and/or poor explanation. This is a cause for much of the confusion regarding the ideas in this theory. I find it best to build an idea from other ideas a student (or reader) already knows, which is a philosophy I've used in writing this book. We've spent a lot of time focused on coordinate systems and diagrams. This also seems like a good place to start with this.

A major implication of special relativity is that *time* deserves as much attention as *space*. Diagrammatically, that means we'll need to include it in the coordinate system resulting in a four-dimensional **spacetime**. With the new idea of a spacetime comes some new terminology:

- **Spacetime diagram** - A diagram which includes both space and time.
- **Event** - A point in spacetime designated by four coordinates, $(ct, x, y, z)$. Essentially, it's a place and time for some phenomenon.
- **Separation** - The straight line connecting two events in spacetime. The word "distance" is improper with a time component involved.
- **World line** - The path taken by a particle/object in spacetime. The word "trajectory" is improper with a time component involved.

In Figure 7.2, we see two objects initially located at events 1 and 3. At some time $\Delta t$ later, they are at events 2 and 4, respectively, where they are now closer in space. The line between events 1 and 2 is labeled $\Delta s$, which represents the world line of that object. The length of this world line is **spacetime invariant** (i.e. it doesn't change under coordinate transformations).

### Line Element

The best tools we have to describe a space are given in Section 6.4. However, we have to be very careful when we incorporate time. First, time is not measured in the same units as space, so a conversion factor of $c$ (the speed of light) appears. Secondly, by observation, we see that time behaves a little

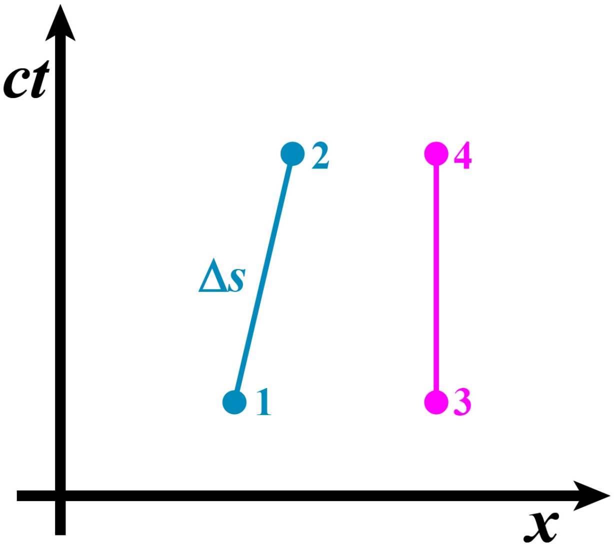

Figure 7.2: This is a spacetime diagram where the horizontal axis, $x$, represents space ($y$ and $z$ are suppressed for simplicity) and the vertical axis, $ct$ represents time measured in spatial units ($c = 299,792,458$ m/s is like a unit conversion between meters and seconds).

differently than space. It behaves *oppositely* to space, so a negative sign also appears. Keeping all this in mind, the Cartesian line element is now

$$ds^2 = -c^2\,dt^2 + dx^2 + dy^2 + dz^2\,, \tag{7.2.1}$$

which is similar to Eq. 6.4.1. Similar to Eq. 6.4.2, we can write

$$ds^2 = -c^2\,dt^2 + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\,d\phi^2\,, \tag{7.2.2}$$

which is the line element in spherical coordinates. We have simply replaced the spatial terms, with the appropriate dimension-3 line element.

Formulating the mathematics of special relativity in this way was not initially done by Einstein. Einstein's methods involved simple algebra and thought experiments ("Gedankenexperimente" as he called them). He was self-admittedly poor with advanced math. In 1908, Hermann Minkowski generalized Einstein's work with tensor analysis (described in Chapter 6). This is why the space described in this chapter is sometimes called the **Minkowski space**.

Since the labeled world line in Figure 7.2 is straight (true of all world lines in IRFs), we can write it as $(\Delta s)^2 = -c^2\,(\Delta t)^2 + (\Delta x)^2$, which looks a lot like the Pythagorean theorem by no coincidence. The negative sign on the time component provides some interesting consequences. One consequence is the square of the separation, $(\Delta s)^2$, is not restricted to positive values. We can use this fact to categorize separations in spacetime.

- If $(\Delta s)^2 < 0$, then the two events have a **time-like** separation meaning the time component dominates. All events on world lines showing the motion of massive objects have this kind of separation (considering the large value of $c$). These world lines are often referred to as time-like world lines.

- If $(\Delta s)^2 = 0$, then the two events have a **light-like** separation because these world lines show the motion of light (and any other massless particle). It is sometimes called a **null** separation because the separation is zero.

- If $(\Delta s)^2 > 0$, then the two events have a **space-like** separation meaning the space component dominates. These two events are considered non-interactive. For an object to travel on a space-like world line, it would require speeds faster than $c$. For this reason, it is unlikely the motion of anything could be represented by a space-like world line.

From a mathematical standpoint, you could write the time component as an imaginary number since

$$\sqrt{-c^2 (\Delta t)^2} = \sqrt{-1}\, c\Delta t = ic\Delta t.$$

This isn't traditionally done. However, it's mathematically consistent and may be useful under circumstances when you're dealing with the components by themselves rather than in a line element.

## Metric Tensor

We can also write something like Eq. 6.4.3 to generalize the line element. The result is

$$\boxed{ds^2 = g_{\alpha\delta}\, dx^\alpha\, dx^\delta}, \tag{7.2.3}$$

where the use of greek indices indicates four dimensions and repeated indices indicates a summation. Remember to distinguish between exponents of 2

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